

Instantons confined by monopole strings

Muneto Nitta¹

¹ *Department of Physics, and Research and Education Center for Natural Sciences,
Keio University, Hi-yoshi 4-1-1, Yokohama, Kanagawa 223-8521, Japan*

(Dated: March 26, 2013)

Abstract

It is known that monopoles can be confined by vortex-strings in $d = 3 + 1$ while vortices can be confined by domain-lines in $d = 2 + 1$. Here, as a higher dimensional generalization of these, we show that Yang-Mills instantons can be confined by monopole-strings in $d = 4 + 1$. We achieve this by putting the system into the Higgs phase in which the configuration can be constructed inside a non-Abelian vortex sheet.

PACS numbers: 11.27.+d, 14.80.Hv, 11.30.Pb, 12.10.-g

I. INTRODUCTION

Quark confinement is one of the most challenging problems in modern high energy physics. Color electric fluxes are expelled or squeezed in the QCD vacuum, and quarks are confined by electric fluxes. A quark and an anti-quark are connected by a color electric flux, constituting a meson bound state. This configuration shows a linear potential with respect to the distance between a well-separated quark and anti-quark [1]. This can be understood in a dual superconductor picture, in which magnetic fluxes are expelled or squeezed in dual Higgs vacuum, as in conventional superconductors. In the Higgs vacuum where the $U(1)$ gauge symmetry is broken, a magnetic monopole and an anti-monopole are confined by a magnetic flux, that is, a magnetic vortex [Fig. 1(c)] [2]. In the standard model of the electroweak unification, the electroweak Z -string ends on a Nambu monopole [3, 4]. In these models, a long vortex is merely metastable and can decay through quantum tunneling by creating a pair of a monopole and an anti-monopole [5]. These configurations were also studied in cosmology to resolve the monopole problem [6, 7]; monopoles and anti-monopoles annihilate in pairs pulled by the tension of a vortex-string connecting them, if the $U(1)$ gauge symmetry is spontaneously broken in the Higgs phase at some stage in the early Universe. Recently, these configurations have been generalized to a non-Abelian monopole confined by non-Abelian vortices to discuss non-Abelian duality in supersymmetric QCD [8]. On the other hand, stable bound states of a magnetic monopole and vortices have been found in supersymmetric QCD [9–13]. In this case, one magnetic monopole is attached by two vortices from both of its sides [Fig. 1(d)]. When the tensions of the both vortices are equal, the total configuration is stable. This is indeed the case of supersymmetry since the vortices are Bogomol’nyi-Prasad-Sommerfield (BPS) states whose tension is proportional to the topological charge. These types of monopoles can be realized as kinks inside a vortex-string [9–13]. These monopole-vortex complexes are a story in $d = 3 + 1$ dimensions, and here we extend them to confined solitons in diverse dimensions.

In $d = 2 + 1$ dimensions, a vortex is a particle-like soliton. In the vortex confinement, a vortex and an anti-vortex are connected by a domain wall or domain line, which is string-like in this dimensionality [Fig. 1(a)]. This was used in a seminal paper by Polyakov for a proof of the confinement of compact QED in $d = 2 + 1$ dimensions [14]. A “quark” and “anti-quark” in QED are mapped by a duality to a vortex and an anti-vortex, respectively. And then, a

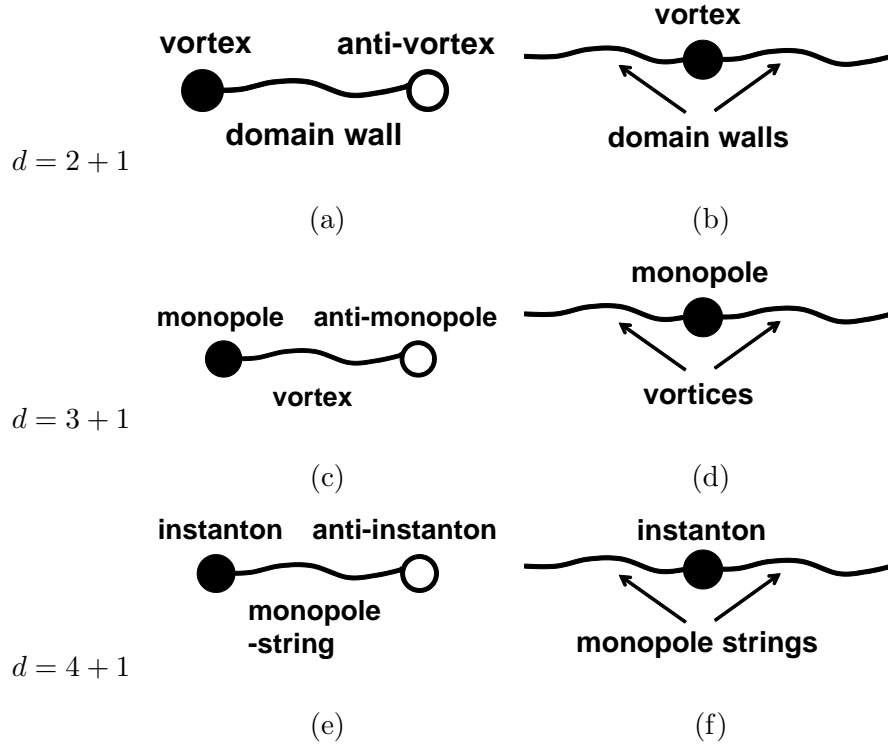


FIG. 1: Soliton confinements. (a) Unstable and (b) stable vortex confinements by domain lines, (c) unstable and (d) stable monopole confinements by vortex-strings, and (e) unstable and (f) stable instanton confinements by monopole-strings.

domain wall generated by a quantum effect connects them to result in the confinement. This is also promoted to 3+1 dimensions with one compactified dimension in an attempt to show confinement in 3+1 dimensions [15]. Vortices connected by a domain wall are also known in several condensed matter systems such as multi-gap superconductors [16, 17] and multi-component Bose-Einstein condensates with a Rabi coupling [18, 19]. In the second (stable) type of vortex confinement, a vortex is attached by domain walls with the same tension from both of its sides [Fig. 1(b)] [20–22]. This confined vortex can be realized as a kink inside a domain wall. In condensed matter physics, this structure appears as a Bloch line in a Bloch wall in magnetism [23], half-quantized vortices trapped in a chiral domain wall in chiral p-wave superconductors [24], and a Mermin-Ho vortex within a domain wall in superfluid ^3He (see Fig. 16.9 of Ref. [25]). Also, a magnetic flux is confined as a Josephson vortex in a Josephson junction of two superconductors [26], where the junction can be identified with an infinitely heavy domain wall. These configurations can be linearly extended into

$d = 3 + 1$ dimensions, where vortex-strings are attached at the edges of a domain wall, that is, a domain wall is bounded by vortex-strings [7, 27]. In the first (unstable) type of vortex confinement, a domain wall is metastable and can decay quantum mechanically, creating a two dimensional hole bounded by a closed vortex-string [5, 18].

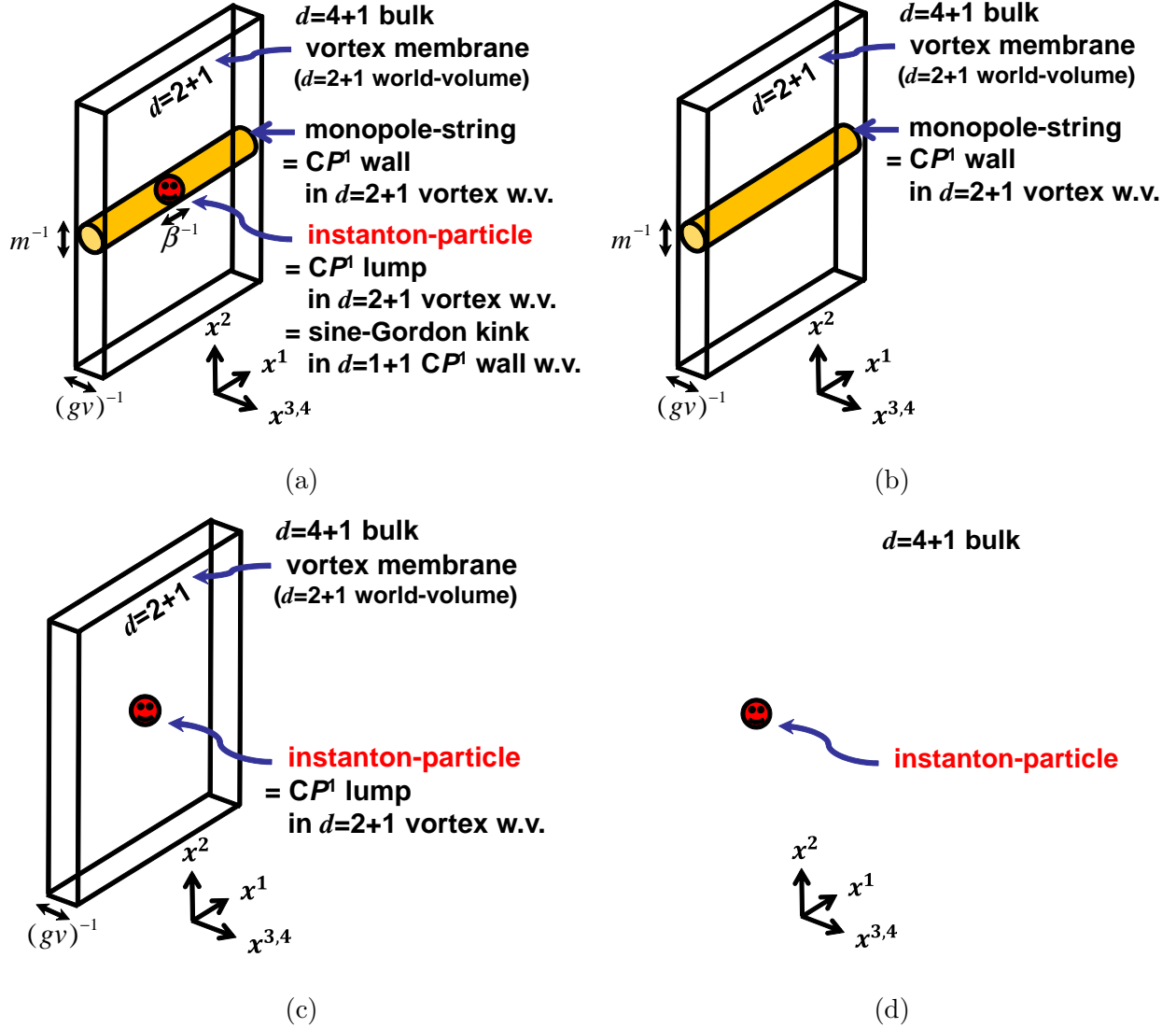


FIG. 2: (a) Original configuration, (b) $\beta \rightarrow 0$, (c) $\beta, m \rightarrow 0$ with $\beta/m^2 = \text{fixed}$, (d) $\beta, v, m \rightarrow 0$ with $\beta/m^2 = \text{fixed}$.

In this paper, we discuss a $d = 4 + 1$ dimensional analog of these confined solitons. In this dimensionality, Yang-Mills instantons are particle-like and 't Hooft-Polyakov monopoles [29] are string-like. Therefore, it is natural to expect that a Yang-Mills instanton is confined by monopole-strings as in Figs. 1(e) and 1(f). Here, we will show that this is indeed the

case in a certain situation by focusing on the second (stable) type of confinement, that is, instantons can be realized as sine-Gordon kinks on a monopole-string. However, it is not straightforward to show this. A 't Hooft-Polyakov monopole has a $U(1)$ modulus related to unbroken $U(1)$ gauge symmetry in the vacuum. This fact implies that a phase kink winding around the $U(1)$ modulus along the string is unstable against expansion, and that the kink on the string as an instanton is diluted along the string. In order to stabilize it, we need to introduce a potential term along the $U(1)$ modulus. However, since this $U(1)$ modulus comes from part of an $SU(2)$ gauge group, it is difficult to have a potential term along the $U(1)$ modulus. The idea to overcome this problem here is to put the theory into the Higgs phase by introducing Higgs fields in the fundamental representation with the vacuum expectation values (VEVs). Non-Abelian vortices exist in this theory [8, 11, 12, 28], and a monopole-string becomes a domain wall inside a non-Abelian vortex [9, 11, 12]. We then introduce additional couplings among the Higgs fields, which we call the non-Abelian Josephson term. In this setup, we show that a sine-Gordon potential term is induced on the effective theory along the monopole-string and that a sine-Gordon kink on the monopole-string as shown in Fig. 2(a) indeed carries the instanton charge.

In Sec. II, we give our model, realize a monopole-string inside a non-Abelian vortex sheet, and construct the effective theory on the monopole-string. In Sec. III, we deform the model by a non-Abelian Josephson term, find the monopole effective Lagrangian that becomes the sine-Gordon model, and construct instanton beads on a monopole-string as sine-Gordon kinks. We discuss various limits in Fig. 2. Section IV is devoted to a summary and discussion.

II. MONOPOLE-STRINGS CONFINED BY VORTEX MEMBRANES

We consider 't Hooft-Polyakov monopole-strings [29] in the BPS limit in a $U(2)$ gauge theory in $d = 4 + 1$ dimensions. We put the system into the Higgs phase, where magnetic fluxes from a monopole are squeezed into vortices [9], which are membranes in $d = 4 + 1$ dimensions. The Lagrangian which we consider in $d = 5 + 1$ dimensions is given by ($A, B = 0, 1, 2, 3, 4$)

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4g^2}\text{tr } F_{AB}F^{AB} + \frac{1}{2g^2}\text{tr } (D_A\Sigma)^2 + \text{tr } D_A H^\dagger D^A H - V, \\ V &= g^2\text{tr } (HH^\dagger - v^2\mathbf{1}_2)^2 + \text{tr } [H(\Sigma - M)^2 H^\dagger],\end{aligned}\tag{1}$$

with a two by two matrix of complex scalar fields H with $D_A H = \partial_A H - ig A_A H$ and a two by two real matrix of adjoint scalar fields Σ with $D_A \Sigma = \partial_A \Sigma - ig [A_A, \Sigma]$. In $d = 4 + 1$ dimensions, this Lagrangian can be made $\mathcal{N} = 2$ supersymmetric (*i.e.*, with eight supercharges) by suitably adding fermions. The mass matrix is given by $M = \text{diag.}(m_1, m_2)$, with $m_1 > m_2$ and $m_1 - m_2 \equiv m$. The constant v^2 giving a VEV to H is called the Fayet-Iliopoulos parameter in the context of supersymmetry. In the limit of vanishing v^2 , the system goes back into the unbroken phase and H decouples in the vacuum. In the massless limit $m = 0$, the Lagrangian (1) enjoys the $SU(2)_F$ flavor symmetry which acts on H from its right. In this case, the system is in the so-called color-flavor locked vacuum $H = \text{diag.}(v, v)$ and $\Sigma = 0$, in which both the color $U(2)$ and flavor $SU(2)_F$ symmetries are spontaneously broken, with the color-flavor locked symmetry $SU(2)_{C+F}$ remaining. In this case, the model admits a non-Abelian $U(2)$ vortex solution [28], $H = \text{diag.}(f(r)e^{i\theta}, v)$, where (r, θ) are polar coordinates in the x^3 - x^4 plane, where the vortex world-volume has the coordinates (x^0, x^1, x^2) . The transverse width of the vortex is $1/gv$. The vortex solution breaks the vacuum symmetry $SU(2)_{C+F}$ into $U(1)$ in the vicinity of the vortex, and consequently there appear $\mathbf{CP}^1 \simeq U(2)_{C+F}/U(1)$ Nambu-Goldstone modes localized around the vortex. The vortex solutions have the orientational moduli \mathbf{CP}^1 in addition to the translational (position) moduli z . By promoting the moduli to the fields depending on the world-volume coordinates (x^0, x^1, x^2) , the low-energy effective theory of these modes can be constructed to yield the \mathbf{CP}^1 model in $d = 2 + 1$ dimensions. In the presence of mass, *i.e.* $m \neq 0$, the $SU(2)_{C+F}$ symmetry is explicitly broken. Considering a regime $m \ll gv$ of small mass, it induces the mass in the $d = 2 + 1$ dimensional vortex effective theory [9, 10, 28, 30] ($\mu = 0, 1, 2$)

$$\mathcal{L}_{\text{vort.eff.}} = 2\pi v^2 |\partial_\mu z_0|^2 + \frac{4\pi}{g^2} \left[\frac{\partial_\mu u^* \partial^\mu u - m^2 |u|^2}{(1 + |u|^2)^2} \right]. \quad (2)$$

Here $z_0(x^\mu), u(x^\mu) \in \mathbf{C}$ represent the position and orientational moduli (the projective coordinate of \mathbf{CP}^1), respectively. The vacua in the vortex theory are $u = 0$ and $u = \infty$ corresponding to the north and south poles of the target space \mathbf{CP}^1 . This is known as the massive \mathbf{CP}^1 model, which can be made supersymmetric with fermions [32].

A monopole solution can be constructed as a domain wall interpolating the two vacua $u = 0$ and $u = \infty$ [32] in the vortex effective theory (2). We place it perpendicular to the x^2 -coordinate as

$$u_{\text{mono.}}(x^2) = e^{\mp m(x^2 - Y) + i\varphi}, \quad (3)$$

where \mp represents a monopole and an anti-monopole with the width $1/m$. Here, Y and φ are moduli parameters representing the position in the x^2 -coordinate and $U(1)$ phase of the (anti-)monopole. The domain wall tension $E_{\text{wall}} = \frac{4\pi}{g^2} \times m = E_{\text{mono.}}$ coincides with the monopole mass $E_{\text{mono.}}$ and the monopole charge in the bulk theory. Therefore, the wall in the vortex theory is nothing but a monopole-string from the bulk point of view [9] as schematically shown in Fig.2(b). In the limit $v \rightarrow 0$ where the system goes back to an unbroken phase, the vortex disappears and the monopole goes back to the original 't Hooft-Polyakov monopole.

Let us construct the effective theory of the monopole-string by promoting the moduli Y and φ to fields $Y(x^i)$ and $\varphi(x^i)$ ($i = 1, 2$) [30, 31] on the string:

$$\begin{aligned}\mathcal{L}_{\text{mono.eff.}} &= \frac{4\pi}{g^2} \int_{-\infty}^{+\infty} dy \frac{e^{2my}}{(1 + e^{2my})^2} [(\partial_i Y)^2 + (\partial_i \varphi)^2] \\ &= \frac{4\pi}{g^2} \frac{1}{2m} [(\partial_i Y)^2 + (\partial_i \varphi)^2].\end{aligned}\tag{4}$$

We now have a free theory, a sigma model with the target space $\mathbf{R} \times U(1)$. As denoted in the introduction, even if we consider a phase kink winding around the $U(1)$ modulus along the string, it is unstable against expansion and is diluted along the string. In order to stabilize it, we need a potential term which prevents the expansion.

III. INSTANTON BEADS ON A MONOPOLE-STRING

In order to stabilize a phase kink on the monopole-string, we introduce the following deformation term in the original Lagrangian in $d = 4 + 1$:

$$\Delta\mathcal{L} = -\frac{\beta^2}{v^2} \text{tr} (H\sigma_x H^\dagger).\tag{5}$$

We call this term a “non-Abelian Josephson term” [41]. We work in the parameter region $\beta \ll mv$ to treat this term perturbatively and assume the vortex solution is not modified at the leading order. Then, this term induces a potential term in the vortex effective Lagrangian [33]:

$$\Delta\mathcal{L}_{\text{vort.eff.}} = -c\beta^2 D_x.\tag{6}$$

Here, D_x is a so-called moment map, defined by

$$D_x = \frac{u + u^*}{1 + |u|^2},\tag{7}$$

and c is a numerical constant given by the integration

$$c = \sqrt{2}\pi \int_0^\infty dr r(v^2 - f^2) \equiv \frac{\tilde{c}}{g^2} (> 0), \quad (8)$$

with the vortex profile function f . In order to evaluate this integral, we approximate the profile function as $f = gv^2r$ for $r \leq 1/gv$ and $f = v$ for $r \geq 1/gv$. We then get:

$$\tilde{c} \sim \frac{\sqrt{2}\pi}{4} \sim 1.11. \quad (9)$$

Since we are working in the parameter region of small β , we can assume that the monopole solution (3) is not deformed at the leading order. Then, the monopole effective Lagrangian is deformed by

$$\Delta\mathcal{L}_{\text{mono.eff.}} = c\beta^2 \int_{-\infty}^{+\infty} dy \frac{e^{my+i\varphi} + e^{my-i\varphi}}{1 + e^{2my}} = \frac{\pi c\beta^2}{m} \cos \varphi. \quad (10)$$

Finally we arrive at the monopole effective Lagrangian summarized as

$$\begin{aligned} \mathcal{L}_{\text{mono.eff.}} + \Delta\mathcal{L}_{\text{mono.eff.}} &= \frac{4\pi}{g^2} \frac{1}{2m} [(\partial_i Y)^2 + (\partial_i \varphi)^2] + \frac{\pi c\beta^2}{m} \cos \varphi \\ &= \frac{2\pi}{g^2 m} [(\partial_i Y)^2 + (\partial_i \varphi)^2 + \tilde{\beta}^2 \cos \varphi], \end{aligned} \quad (11)$$

with

$$\tilde{\beta}^2 \equiv \frac{1}{2} \tilde{c} \beta^2 \sim 0.555 \beta^2. \quad (12)$$

Here we have used the estimation of c in Eq. (9). The monopole effective Lagrangian in Eq. (11) is the one of the sine-Gordon model with the additional field Y .

Let us construct a sine-Gordon kink. The Bogomol'nyi completion for the energy density is

$$\begin{aligned} \frac{g^2 m}{2\pi} E &= (\partial_i \varphi)^2 + \tilde{\beta}^2 \left(\sin^2 \frac{\varphi}{2} - 1 \right) \\ &= \left(\partial_i \varphi \pm \tilde{\beta} \sin \frac{\varphi}{2} \right)^2 \mp 2\tilde{\beta} \partial_i \varphi \sin \frac{\varphi}{2} - \tilde{\beta}^2 \\ &\geq \frac{g^2 m}{2\pi} |t_{\text{SG}}| - \tilde{\beta}^2 \end{aligned} \quad (13)$$

with the topological charge density

$$t_{\text{SG}} \equiv \frac{\tilde{\beta}}{m} \partial_i \varphi \sin \frac{\varphi}{2} = -\frac{2\tilde{\beta}}{m} \partial_i \left(\cos \frac{\varphi}{2} \right). \quad (14)$$

The inequality is saturated by the BPS equation

$$\partial_i \varphi \pm \tilde{\beta} \sin \frac{\varphi}{2} = 0. \quad (15)$$

For instance, the one-kink solution and its topological charge are

$$\varphi = 4 \arctan \exp \frac{\tilde{\beta}}{4}(x - X) + \frac{\pi}{2}; \quad T_{\text{SG}} = \int dy t_{\text{SG}} = \frac{4\tilde{\beta}}{m}. \quad (16)$$

The width of the kink is $\Delta x \sim 1/\tilde{\beta}$. The energy of the one sine-Gordon kink is

$$E_{\text{SG}} = \frac{2\pi}{g^2 m} T_{\text{SG}} = \frac{8\pi\tilde{\beta}}{g^2 m^2} = 4\sqrt{2}\pi\sqrt{\tilde{c}} \frac{\beta}{g^2 m^2} \sim 18.7 \frac{\beta}{g^2 m^2}. \quad (17)$$

What does the sine-Gordon kink correspond to in the vortex theory and in the bulk? First, one can confirm that k sine-Gordon kinks can be identified with k \mathbf{CP}^1 -lumps with the topological charge $k \in \pi_2(\mathbf{CP}^1)$ [34] in $d = 2 + 1$ dimensional vortex world-volume, by explicitly calculating a lump charge

$$\begin{aligned} T_{\text{lump}} &\equiv \int d^2 x \frac{i(\partial_i u^* \partial_j u - \partial_j u^* \partial_i u)}{(1 + |u|^2)^2} \\ &= \oint dx^i \frac{-i(u^* \partial_i u - (\partial_i u^*) u)}{2(1 + |u|^2)} \\ &= \oint dx^i \frac{|u|^2}{1 + |u|^2} \partial_i \varphi = 2\pi k. \end{aligned} \quad (18)$$

The \mathbf{CP}^1 lumps in the vortex effective theory can be further identified with Yang-Mills instantons in the bulk [10] as can be inferred from the lump energy E_{lump} , coinciding with the instanton energy E_{inst} [10]: $E_{\text{lump}} = \frac{4\pi}{g^2} T_{\text{lump}} = \frac{4\pi}{g^2} \times 2\pi k = \frac{8\pi^2}{g^2} k = E_{\text{inst}}$. Therefore, we conclude that the sine-Gordon kink on the monopole-string corresponds to a Yang-Mills instanton in the bulk point of view, as schematically shown in Fig.2(a). Multiple sine-Gordon kinks give multiple instanton beads on the monopole-string.

To clarify the relations with configurations known before, let us discuss various limits shown in Fig. 2.

1. $\beta \rightarrow 0$ [Fig. 2(b)]: Monopole-string confined by vortex-sheets [9].

The instanton is diluted and eventually disappears because the energy of the sine-Gordon kink becomes zero.

2. $\beta, m \rightarrow 0$ with $\beta/m^2 = \text{fixed} = \frac{\sqrt{2}\pi}{\sqrt{\tilde{c}}}$ [Fig.2(c)]: Instanton particle trapped inside a vortex sheet [10].

The monopole-string disappears while the vortex-sheet and instanton remain. The scaling is determined by requiring the energy of a sine-Gordon kink in Eq. (17) to reduce to the BPS instanton energy $= 8\pi^2/g^2$. The precise value of the constant \tilde{c} is needed to determine the scaling constant.

3. $\beta, v, m \rightarrow 0$ with $\beta/m^2 = \text{fixed} = \frac{\sqrt{2}\pi}{\sqrt{\tilde{c}}}$ [Fig.2(d)]: Bare instanton particle.

Only the instanton remains while the vortex-sheet and monopole-string disappear. The transverse size of the vortex $1/gv$ grows, and the vortex is diluted and disappears from the limit 2.

From these limits, one may think that the simple limit $v \rightarrow 0$ gives an instanton on a monopole-string without a vortex sheet. However, the vacuum expectation value of H vanishes in the limit $v \rightarrow 0$ so that the deformation by β in Eq. (5) does not seem to affect the configuration. The instanton will be diluted along the infinitely long monopole-string. This limit seems to be rather subtle.

In this paper, we have constructed the stable type of instanton confinement in Fig. 1(f). In a model where a monopole-string is unstable, the monopole-string decays through quantum tunneling by creating an instanton and an anti-instanton as shown in Fig. 1(e).

IV. SUMMARY AND DISCUSSION

We have constructed instanton beads on a monopole-string in $d = 4 + 1$, which shows that an instanton is confined by monopole-strings as a higher dimensional generalization of a monopole confined by vortex-strings in $d = 3 + 1$ and a vortex confined by domain-lines in $d = 2 + 1$. We have constructed this configuration inside a non-Abelian vortex sheet by putting the system into the Higgs phase in supersymmetric gauge theory with a deformation term, which we call the non-Abelian Josephson term. A monopole-string has been realized as a domain-line inside the vortex, and we have found that the effective theory on the monopole-string is the sine-Gordon model whose potential is induced by the non-Abelian Josephson term. Then, the instanton has been realized as a sine-Gordon kink in the monopole effective theory. We have discussed several limits of the configuration reducing to previously known configurations such as a confined monopole on the vortex and an instanton trapped inside the vortex.

A necklace of instantons can be constructed by making a closed monopole-string inside a non-Abelian vortex.

It is unclear thus far whether instanton beads on a monopole-string can exist without the help of a vortex. Without the potential on the monopole theory, the instanton charge made of phase kinks will be diluted along the infinitely long monopole-string and disappear. However, if one makes phase kinks on a closed monopole-string, one gets a twisted closed monopole-string where the instanton charge is uniformly distributed along the loop. It is nothing but an instanton [35].

We have studied $U(2)$ gauge theory with two flavors, while a generalization to $U(N)$ gauge group with N flavors can be made. When we consider non-degenerate masses for H , there appear $N - 1$ parallel monopole-strings inside the vortex-sheet as $N - 1$ domain walls [36]. By considering a non-Abelian Josephson term such as $\sum_{a \in \text{root}} \beta_a^2 \text{tr}(HT_a H^\dagger)$, we would have sine-Gordon kinks in multiple monopole-strings, which remain as a future problem. Moreover, if we consider degenerate masses for H in the $U(N)$ case, monopoles become non-Abelian monopoles [13] inside a vortex as non-Abelian domain walls [37–39]. Then, a non-Abelian generalization of sine-Gordon kinks should be considered as instantons on a non-Abelian monopole-string. Generalization to arbitrary gauge groups is also possible as was so for a non-Abelian vortex [40].

Acknowledgements

This work is supported in part by a Grant-in-Aid for Scientific Research (No. 23740198) and by the “Topological Quantum Phenomena” Grant-in-Aid for Scientific Research on Innovative Areas (No. 23103515) from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan.

-
- [1] G. S. Bali: “QCD forces and heavy quark bound states,” *Phys. Rept.* **343**, 1 (2001).
 - [2] Y. Nambu, “Strings, Monopoles and Gauge Fields,” *Phys. Rev. D* **10**, 4262 (1974); S. Mandelstam, “Vortices And Quark Confinement In Nonabelian Gauge Theories,” *Phys. Lett. B* **53**, 476 (1975); S. Mandelstam, “Vortices and Quark Confinement in Nonabelian Gauge Theories,” *Phys. Rept.* **23**, 245 (1976).

- [3] Y. Nambu, “String-Like Configurations in the Weinberg-Salam Theory,” Nucl. Phys. B **130**, 505 (1977).
- [4] M. Eto, K. Konishi, M. Nitta and Y. Ookouchi, “Brane Realization of Nambu Monopoles and Electroweak Strings,” Phys. Rev. D **87**, 045006 (2013) [arXiv:1211.2971 [hep-th]].
- [5] J. Preskill and A. Vilenkin, “Decay of metastable topological defects,” Phys. Rev. D **47**, 2324 (1993).
- [6] P. Langacker and S. -Y. Pi, “Magnetic Monopoles in Grand Unified Theories,” Phys. Rev. Lett. **45**, 1 (1980); G. Lazarides, Q. Shafi and T. F. Walsh, “Cosmic Strings And Domains In Unified Theories,” Nucl. Phys. B **195**, 157 (1982); A. Vilenkin, “Cosmological Evolution Of Monopoles Connected By Strings,” Nucl. Phys. B **196**, 240 (1982); M. Hindmarsh and T. W. B. Kibble, “Beads On Strings,” Phys. Rev. Lett. **55**, 2398 (1985).
- [7] A. Vilenkin and E. P. S. Shellard, *Cosmic Strings and Other Topological Defects*, (Cambridge Monographs on Mathematical Physics), Cambridge University Press (July 31, 2000).
- [8] R. Auzzi, S. Bolognesi, J. Evslin, K. Konishi and A. Yung, “NonAbelian superconductors: Vortices and confinement in N=2 SQCD,” Nucl. Phys. B **673**, 187 (2003) [hep-th/0307287]; R. Auzzi, S. Bolognesi, J. Evslin and K. Konishi, “NonAbelian monopoles and the vortices that confine them,” Nucl. Phys. B **686**, 119 (2004) [hep-th/0312233]; M. Eto, L. Ferretti, K. Konishi, G. Marmorini, M. Nitta, K. Ohashi, W. Vinci and N. Yokoi, “Non-Abelian duality from vortex moduli: A Dual model of color-confinement,” Nucl. Phys. B **780**, 161 (2007) [hep-th/0611313].
- [9] D. Tong, “Monopoles in the higgs phase,” Phys. Rev. D **69**, 065003 (2004) [hep-th/0307302]; M. Shifman and A. Yung, “NonAbelian string junctions as confined monopoles,” Phys. Rev. D **70**, 045004 (2004) [hep-th/0403149]; A. Hanany and D. Tong, “Vortex strings and four-dimensional gauge dynamics,” JHEP **0404**, 066 (2004) [hep-th/0403158].
- [10] M. Eto, Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, “Instantons in the Higgs phase,” Phys. Rev. D **72**, 025011 (2005) [hep-th/0412048]; T. Fujimori, M. Nitta, K. Ohta, N. Sakai and M. Yamazaki, “Intersecting Solitons, Amoeba and Tropical Geometry,” Phys. Rev. D **78**, 105004 (2008) [arXiv:0805.1194 [hep-th]].
- [11] M. Eto, Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, “Solitons in the Higgs phase: The Moduli matrix approach,” J. Phys. A **39**, R315 (2006) [hep-th/0602170].
- [12] M. Shifman and A. Yung, “Supersymmetric Solitons and How They Help Us Understand

- Non-Abelian Gauge Theories,” *Rev. Mod. Phys.* **79**, 1139 (2007) [hep-th/0703267]; “Supersymmetric solitons,” Cambridge, UK: Cambridge Univ. Pr. (2009) 259 p
- [13] M. Nitta and W. Vinci, “Non-Abelian Monopoles in the Higgs Phase,” *Nucl. Phys. B* **848**, 121 (2011) [arXiv:1012.4057 [hep-th]].
- [14] A. M. Polyakov, “Quark Confinement and Topology of Gauge Groups,” *Nucl. Phys. B* **120**, 429 (1977).
- [15] M. Shifman and M. Unsal, “Confinement in Yang-Mills: Elements of a Big Picture,” *Nucl. Phys. Proc. Suppl.* **186**, 235 (2009).
- [16] Y. Tanaka, “Phase Instability in Multi-band Superconductors,” *J. Phys. Soc. Jp.* **70**, 2844 (2001); “Soliton in Two-Band Superconductor,” *Phys. Rev. Lett.* **88**, 017002 (2001); J. Goryo, S. Soma and H. Matsukawa, “Deconfinement of vortices with continuously variable fractions of the unit flux quanta in two-gap superconductors,” *Euro Phys. Lett.* **80**, 17002 (2007).
- [17] Y. Tanaka and T. Yanagisawa, “Chiral Ground State in Three-Band Superconductors,” *J. Phys. Soc. Jpn.* **79** (2010) 114706; T. Yanagisawa, Y. Tanaka, I. Hase, and K. Yamaji, “Vortices and Chirality in Multi-Band Superconductors,” *J. Phys. Soc. Jpn.* **81** (2012) 024712; M. Nitta, M. Eto, T. Fujimori and K. Ohashi, “Baryonic Bound State of Vortices in Multi-component Superconductors,” *J. Phys. Soc. Jap.* **81**, 084711 (2012) [arXiv:1011.2552 [cond-mat.supr-con]].
- [18] D. T. Son, and M. A. Stephanov, “Domain walls in two-component Bose-Einstein condensates,” *Phys. Rev.* **A65**, 063621 (2002) [cond-mat/0103451].
- [19] K. Kasamatsu, M. Tsubota and M. Ueda, “Vortex Molecules in Coherently Coupled Two-Component Bose-Einstein Condensates,” *Phys. Rev. Lett.* **93**, 250406 (2004); M. Eto and M. Nitta, “Vortex trimer in three-component Bose-Einstein condensates,” *Phys. Rev. A* **85**, 053645 (2012) [arXiv:1201.0343 [cond-mat.quant-gas]].
- [20] A. Ritz, M. Shifman and A. Vainshtein, “Enhanced worldvolume supersymmetry and intersecting domain walls in N=1 SQCD,” *Phys. Rev. D* **70**, 095003 (2004) [hep-th/0405175]; R. Auzzi, M. Shifman and A. Yung, “Domain Lines as Fractional Strings,” *Phys. Rev. D* **74**, 045007 (2006) [hep-th/0606060]; S. Bolognesi, “Strings inside walls in N=1 super Yang-Mills,” *J. Phys. A* **42**, 195404 (2009) [arXiv:0710.5198 [hep-th]].
- [21] M. Eto, Y. Isozumi, M. Nitta and K. Ohashi, “1/2, 1/4 and 1/8 BPS equations in SUSY Yang-Mills-Higgs systems: Field theoretical brane configurations,” *Nucl. Phys. B* **752**, 140

- (2006) [hep-th/0506257].
- [22] M. Nitta, “Josephson vortices and the Atiyah-Manton construction,” *Phys. Rev. D* **86**, 125004 (2012) [arXiv:1207.6958 [hep-th]].
 - [23] C. W. Chen, “Magnetism and metallurgy of soft magnetic materials,” Dover Pubns (1977); A. P. Malozemoff, J. C. Slonczewski, “Magnetic domain walls in bubble materials,” Academic Press (New York) (1979).
 - [24] J. Garaud and E. Babaev, “Skyrmionic state and stable half-quantum vortices in chiral p-wave superconductors,” *Phys. Rev. B* **86**, 060514 (2012).
 - [25] G. E. Volovik, *The Universe in a Helium Droplet*, Clarendon Press, Oxford (2003).
 - [26] A. V. Ustinov, “Solitons in Josephson junctions,” *Physica D* **123**, 315329 (1998).
 - [27] T. W. B. Kibble, G. Lazarides and Q. Shafi, “Walls Bounded by Strings,” *Phys. Rev. D* **26**, 435 (1982); A. Vilenkin and A. E. Everett, “Cosmic Strings and Domain Walls in Models with Goldstone and PseudoGoldstone Bosons,” *Phys. Rev. Lett.* **48**, 1867 (1982).
 - [28] A. Hanany and D. Tong, “Vortices, instantons and branes,” *JHEP* **0307**, 037 (2003) [hep-th/0306150]; M. Eto, Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, “Moduli space of non-Abelian vortices,” *Phys. Rev. Lett.* **96**, 161601 (2006) [hep-th/0511088]; M. Eto, K. Konishi, G. Marmorini, M. Nitta, K. Ohashi, W. Vinci and N. Yokoi, “Non-Abelian Vortices of Higher Winding Numbers,” *Phys. Rev. D* **74**, 065021 (2006) [hep-th/0607070]; M. Eto, K. Hashimoto, G. Marmorini, M. Nitta, K. Ohashi and W. Vinci, “Universal Reconnection of Non-Abelian Cosmic Strings,” *Phys. Rev. Lett.* **98**, 091602 (2007) [hep-th/0609214].
 - [29] G. ’t Hooft, “Magnetic Monopoles In Unified Gauge Theories,” *Nucl. Phys. B* **79**, 276 (1974); A. M. Polyakov, “Particle spectrum in quantum field theory,” *JETP Lett.* **20**, 194 (1974) [*Pisma Zh. Eksp. Teor. Fiz.* **20**, 430 (1974)].
 - [30] M. Eto, Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, “Manifestly supersymmetric effective Lagrangians on BPS solitons,” *Phys. Rev. D* **73**, 125008 (2006) [hep-th/0602289].
 - [31] N. S. Manton, “A Remark on the Scattering of BPS Monopoles,” *Phys. Lett. B* **110**, 54 (1982).
 - [32] E. R. C. Abraham and P. K. Townsend, *Phys. Lett. B* **291**, 85 (1992); *Phys. Lett. B* **295**, 225 (1992); M. Arai, M. Naganuma, M. Nitta and N. Sakai, “Manifest supersymmetry for BPS walls in N=2 nonlinear sigma models,” *Nucl. Phys. B* **652**, 35 (2003) [hep-th/0211103]; “BPS wall in N=2 SUSY nonlinear sigma model with Eguchi-Hanson manifold,” In *Arai, A. (ed.) et al.: A garden of quanta* 299-325 [hep-th/0302028].

- [33] M. Eto, M. Nitta and N. Yamamoto, “Instabilities of Non-Abelian Vortices in Dense QCD,” *Phys. Rev. Lett.* **104**, 161601 (2010) [arXiv:0912.1352 [hep-ph]].
- [34] A. M. Polyakov and A. A. Belavin, “Metastable States of Two-Dimensional Isotropic Ferromagnets,” *JETP Lett.* **22**, 245 (1975) [*Pisma Zh. Eksp. Teor. Fiz.* **22**, 503 (1975)].
- [35] M. Nitta, “Defect formation from defect–anti-defect annihilations,” *Phys. Rev. D* **85**, 101702 (2012) [arXiv:1205.2442 [hep-th]].
- [36] Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, “Construction of non-Abelian walls and their complete moduli space,” *Phys. Rev. Lett.* **93**, 161601 (2004) [hep-th/0404198]; Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, “Non-Abelian walls in supersymmetric gauge theories,” *Phys. Rev. D* **70**, 125014 (2004) [hep-th/0405194]; Y. Isozumi, M. Nitta, K. Ohashi and N. Sakai, “All exact solutions of a 1/4 Bogomol’nyi-Prasad-Sommerfield equation,” *Phys. Rev. D* **71**, 065018 (2005) [hep-th/0405129]; M. Eto, Y. Isozumi, M. Nitta, K. Ohashi, K. Ohta and N. Sakai, “D-brane construction for non-Abelian walls,” *Phys. Rev. D* **71**, 125006 (2005) [hep-th/0412024].
- [37] M. Shifman and A. Yung, “Localization of nonAbelian gauge fields on domain walls at weak coupling (D-brane prototypes II),” *Phys. Rev. D* **70**, 025013 (2004) [hep-th/0312257].
- [38] M. Eto, M. Nitta, K. Ohashi and D. Tong, “Skyrmions from instantons inside domain walls,” *Phys. Rev. Lett.* **95**, 252003 (2005) [hep-th/0508130].
- [39] M. Eto, T. Fujimori, M. Nitta, K. Ohashi and N. Sakai, “Domain walls with non-Abelian clouds,” *Phys. Rev. D* **77**, 125008 (2008) [arXiv:0802.3135 [hep-th]].
- [40] M. Eto, T. Fujimori, S. B. Gudnason, K. Konishi, M. Nitta, K. Ohashi and W. Vinci, “Constructing Non-Abelian Vortices with Arbitrary Gauge Groups,” *Phys. Lett. B* **669**, 98 (2008) [arXiv:0802.1020 [hep-th]].
- [41] The term in Eq. (6) can be rewritten as $cv^2\beta^2\phi^{*1}\phi^2$ in terms of homogeneous coordinates (ϕ^1, ϕ^2) of \mathbf{CP}^1 with the identification $(\phi^1, \phi^2) \sim e^{i\alpha}(\phi^1, \phi^2)$ and the constraint $|\phi^1|^2 + |\phi^2|^2 = 1$. This term is a Josephson term appearing in a Josephson junction of two superconductors with two condensates ϕ^1 and ϕ^2 . In this sense, the term in Eq. (5) can be called the non-Abelian Josephson term.